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# **NASA TECHNICAL MEMORANDUM 109023**

## **PREDICTION OF STABLE TEARING OF 2024-T3 ALUMINUM ALLOY USING THE CRACK-TIP OPENING ANGLE APPROACH**

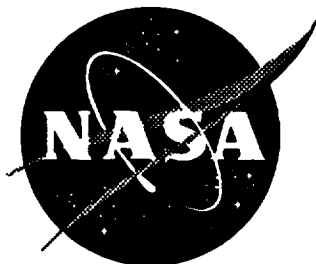
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Hampton, Virginia 23681-0001**

(NASA-TM-109023) PREDICTION OF  
STABLE TEARING OF 2024-T3 ALUMINUM  
ALLOY USING THE CRACK-TIP OPENING  
ANGLE APPROACH (NASA) 24 p



## Abstract

In this study, the crack-tip opening angle (CTOA) approach was incorporated into a damage growth finite element program, MADGIC (Micromechanics Analysis and Damage Growth In Composites), and was used to predict stable tearing in a middle-crack tension 2024-T3 aluminum alloy specimen. The MADGIC code is a displacement based finite element program implemented with an incremental elastic-plastic algorithm used to model elastic-plastic behavior and a nodal splitting and nodal force relaxation algorithm used to generate crack surfaces. Predictions of the applied stress as a function of crack extension and applied stress as a function of load-line displacement were in good agreement with experiments and with similar predictions made using an existing finite element program, ZIP2D. In addition, path integrals, namely, the J-integral and T\*-integral, were also evaluated and compared with the CTOA approach. There appears to be a weak relationship between the CTOA and the T\*-integral evaluated on a specific integration path during crack extension beyond maximum applied stress. This study further verifies that the CTOA can be used as an effective elastic-plastic fracture mechanics parameter to predict crack growth.

## Background

Over the past two decades, numerical methods, such as finite element analyses, have become firmly established as effective tools to solve complex fracture mechanics problems. With improved computer efficiencies, further advances are being realized particularly in elastic-plastic fracture mechanics approaches. Significant efforts have been made to study the parameters characterizing elastic-plastic fracture utilizing numerical methods [1-10]. The two most widely recognized and accepted approaches to characterize elastic-plastic fracture are path integral (J and T\*) approaches [1,8,9] and the crack-tip opening displacement (CTOD) or angle (CTOA) approaches [3,5,7,8]. Both types of approaches have been the subject of much debate.

One of the first path integrals, the J-integral, was introduced by Rice [11] and is defined as a path-independent line integral enclosing the crack-tip:

$$J = \int_{\Gamma} (w n_x - T_i \frac{du_i}{dx_1}) ds \quad (1)$$

where  $\Gamma$  is the integration path taken counterclockwise,  $w$  is the strain energy density,  $n_x$  is the component of the unit normal to the path  $\Gamma$  in the  $X$  direction,  $T_i$  is the outward normal traction vector on the path  $\Gamma$ ,  $u_i$  is the displacement vector, and  $ds$  is the line element on the path  $\Gamma$  as shown in Figure 1. The J-integral is based on deformation theory of plasticity and thus is strictly defined for linear or nonlinear elastic solids. As long as a crack remains stationary and the load is monotonically increasing, deformation theory will accurately model plasticity behavior. The J-integral is a valid parameter to control the onset of cracking. However, for growing cracks where regions of elastic unloading and non-proportional plastic flow are embedded in the crack-tip vicinity, plasticity behavior is not properly modeled by deformation theory. This poses restrictions on the J-integral when used to dictate crack growth. Conditions for J-controlled crack growth have been defined which are analogous to small scale yielding conditions for linear elastic fracture mechanics [12]. The existence of a material J-resistance curve which is independent of specimen configuration occurs when the crack extension length and the region of non-proportional loading are small compared to the region of J-dominance, Figure 1. For fully yielded conditions, J-controlled crack growth is valid when [12]:

$$\frac{b}{J_{Ic}} \frac{dJ}{da} = \omega \gg 1 \quad (2)$$

where  $b$  is the net section ligament of the specimen. The smallest value of  $\omega$  which insures J-controlled crack growth depends on the specimen configuration and the strain hardening characteristics of the material.

The plastic properties of metals are typically modeled according the following power law function:

$$\bar{\sigma} = \bar{\sigma}_1 (\bar{\epsilon}_p)^n \quad (3)$$

where  $\bar{\sigma}$  is the equivalent stress,  $\bar{\sigma}_1$  is the equivalent stress at unit strain,  $\bar{\epsilon}_p$  is the equivalent plastic strain, and  $n$  strain hardening parameter. For middle-crack tension specimens having a value of  $n$  approximately 10, the value of  $\omega$  should be greater than or equal to 100 for valid J-controlled crack growth [13]. When this condition is not satisfied, the J-resistance curve is not

independent of specimen configuration and, thus, the J-integral approach losses both generality and effectiveness. Other parameters, such as T\*-integral or CTOA, are needed when conditions for J-controlled crack growth not satisfied.

The T\*-integral was introduced by Atluri et. al [9,14-15] and is defined as the energy flux to the crack-tip region of a small radius,  $\zeta$ . Analogous to the J-integral, the T\*-integral is a localized path integral enclosing the crack-tip [15]:

$$T^* = \int_{\Gamma_{\zeta}} (w^* n_x - T_i \frac{du_i}{dx_1}) ds \quad (4)$$

There are two major differences between the J-integral and the T\*-integral. First, the J-integral is based on deformation theory of plasticity while the T\*-integral is based on incremental theory of plasticity. Accordingly, the strain energy density,  $w$ , in the J-integral (Equation 1) is redefined as the total accumulated increments of the stress working density,  $w^*$ , in the T\*-integral (Equation 4). The second difference is that the integration path is fixed for the J-integral while a moving integration path,  $\Gamma_{\zeta}$ , which encircles the crack-tip as well as the crack wake is used for the T\*-integral, Figure 2. Figure 2 shows the integration path before and after an increment in crack extension. Prior to any crack extension, Figure 2a, the integration path is a circle of radius  $\zeta$  from the crack-tip. At this point, the J-integral and T\*-integral are equivalent. As the crack extends, the circular path of integration of radius  $\zeta$  translates and elongates with the crack-tip as shown in Figure 2b. Values of the T\*-integral and J-integral deviate during crack growth.

The work reported in references 14 and 15 demonstrated the potential of the T\*-integral as a versatile elastic-plastic fracture mechanics parameter. It was shown that the values of both the CTOA and T\*-integral approached constant levels while the J-integral values continuously increased during stable crack growth in compact tension specimens of A508 and A533-B steel [15]. However, it should be pointed out that the T\*-integral has a computational dependence on the appropriate definition of the radius,  $\zeta$ . This limitation is apparently an artifact of mesh refinement. For small  $\zeta$  values, numerical errors can be introduced in the calculation of the T\*-integral and, therefore, a finer a mesh is required. The smallest acceptable value of  $\zeta$  depends on

the mesh refinement near the crack-tip. The value of  $\zeta$  must be small enough to capture the salient features of the crack-tip fields and large enough to minimize numerical errors due to mesh refinement. Thus, care must be taken when defining the value of  $\zeta$ . As a rule-of-thumb, the value of  $\zeta$  should be approximately 0.25 to 0.50 the size of the plastic zone at crack initiation [15].

The CTOD-CTOA approach was one of the first methods used in elastic-plastic fracture mechanics studies [16]. The CTOD and CTOA are fracture characterizing parameters representing a unique measure of the crack-tip strain field. It was shown by Newman et. al [10] that a CTOA criterion can be used to determine stable tearing in 2024-T3 aluminum alloy. In that study, several specimen geometries were tested and in all cases, stable tearing occurred at nearly a constant value of CTOA. A high resolution, long focal length microscope system with image acquisition capabilities was used to measure the CTOA. The image in Figure 3 illustrates the measurements of CTOA made at several distances behind the crack-tip. The CTOA as a function of the crack extension is also shown in Figure 3 for a middle-crack tension specimen subjected to quasi-static loading in a strain control mode. In general, the CTOA values initially decreased and then approached a constant value after crack extensions greater than the sheet thickness. The higher values of CTOA at the low crack extension values were attributed to three dimensional effects of crack tunneling and some of these effects were modeled using a three dimensional finite element analysis [10]. Once the crack extended beyond the transition region (flat to slant crack growth), stable tearing in a shear mode occurred and the CTOA were nearly a constant value,  $\psi_c = 6.1^\circ$ .

The CTOD-CTOA approach is ideally suited for numerical modeling of stable crack growth and instability during elastic-plastic fracture [3,5,7,8]. The CTOA approach was incorporated into a crack growth finite element program, ZIP2D, and was used to predict stable tearing in 2024-T3 aluminum having several specimen configurations [10]. The ZIP2D code is a two dimensional, elastic-plastic finite element program implemented with crack growth and crack closure features [17]. Using  $\psi_c = 6.1^\circ$  in the ZIP2D code, the applied stress as a function of crack extension and applied stress as a function of load-line displacement were accurately predicted in several crack configurations made of 2024-T3 aluminum alloy.

In this study, the CTOA approach was incorporated into a new damage growth finite element program, MADGIC (Micromechanics Analysis and Damage Growth In Composites), and was used to predict stable tearing in a middle-crack tension 2024-T3 aluminum alloy specimen. The MADGIC code [18] is a displacement based finite element program implemented with an incremental elastic-plastic algorithm and crack growth features. The major differences between the MADGIC code and the ZIP2D code are the algorithms used to generate crack surfaces and the element types employed. The MADGIC code uses a nodal splitting and nodal force relaxation algorithm [18] to generate crack surfaces while ZIP2D uses node coupled spring force relaxation algorithm [17]. In addition, the elements used in the MADGIC code analysis were 9-noded isoparametric elements (with 2 by 2 numerical integration) while in the ZIP2D code analysis, 3-noded constant strain triangular elements were employed. Both finite element codes have advantages; the crack path need not be preselected in the MADGIC code whereas crack closure effects are modeled in the ZIP2D code. The primary purpose of this study was to further evaluate the CTOA criterion by comparing the results from two finite element approaches using fundamentally different crack surface generation algorithms and element types. In addition, the correlation that may exist between the various elastic-plastic fracture mechanics parameters, namely, the CTOA, J-integral and  $T^*$ -integral was investigated.

### **Numerical Methodology**

In this study, the CTOA approach was implemented in the MADGIC code and was used to predict stable tearing in 2024-T3 aluminum alloy. An algorithm to calculate the path integrals, namely, the J-integral and  $T^*$ -integral, was also incorporated in the code so that the correlation between the CTOA and the path integrals can be evaluated. A brief description of the finite element model used and the elastic-plastic fracture mechanics parameters calculated follows.

### **Finite Element Model**

Stable tearing in a middle cracked 2024-T3 aluminum alloy specimen ( $2a/W = 0.35$ ) subjected to quasi-static tensile loading under displacement control was simulated using the MADGIC code implemented with a critical CTOA criterion. The specimen geometry is shown in Figure 4 along with the associated finite element representation. Due to symmetry in the applied loading and the specimen geometry, only one quadrant was analyzed. The entire finite element mesh consisted of 450 elements and 1889 nodes. Nine-noded isoparametric elements with  $2 \times 2$  Gauss numerical integration were used. The crack-tip area had a refined and uniform mesh configuration where the dimension of the smallest element was 0.01875 inches x 0.01875 inches which also corresponded to the smallest element size used by Newman et. al [10]. Plane stress conditions were assumed in this study. In addition, the elastic-plastic properties of the 2024-T3 aluminum used in the analysis were identical to those in reference 10.

Using the mesh shown in Figure 4, a preliminary linear-elastic analysis was conducted to evaluate the discretization of the mesh. Figure 5 shows the variation of the longitudinal stresses through the net section (i.e.  $\sigma_{yy}(x,0)$ ) calculated using the MADGIC code and calculated using the handbook [20] value of the stress intensity factor. As shown in this figure, good agreement was obtained between the elastic analyses. The finite element mesh used was assumed to capture the essential features of the crack-tip singularity fields during elastic-plastic fracture.

### **Elastic-Plastic Fracture Parameters. Path Integrals and CTOA:**

Both the path integral and CTOA approaches were incorporated into the MADGIC code. The path integral algorithm employed was similar to that outlined in [19]. Using the mesh shown in Figure 4, a preliminary linear-elastic analysis was conducted in order to evaluate the path integral algorithm. Calculations of the J-integral are shown in Figure 6a using several paths passing through the elements integration points as illustrated in Figures 6b and 6c. Also shown in this figure is the linear-elastic solution based on the handbook [20] value of the stress intensity factor for a middle-crack tension specimen. The J-integral values were nearly constant and in excellent



agreement with the handbook value. The far-field J-integral was calculated for a fixed integration path, Figure 6c. The T\*-integral was calculated using several values of  $\zeta$  as shown in Figure 6b. Note that the T\*-integral is actually calculated on rectangular paths passing through the element integration points. The ring of elements defining integration paths for the T\*-integral elongates during crack growth as shown schematically in Figure 2.

The CTOA approach was implemented to determine crack extension using the following definition for CTOA:

$$\psi = \tan^{-1} (\delta/d) \quad (5)$$

In this equation,  $\psi$  is the CTOA, and  $\delta$  is the difference in the Y displacements of the nodes located distance,  $d$ , directly behind the crack-tip as shown in Figure 7. The distance,  $d = 0.01875$  inches, is equivalent to one element length. The critical value of CTOA was assumed to be  $\psi_c = 6.1^\circ$  which was the typical value measured during stable tearing in 2024-T3 aluminum alloy [10].

A uniform displacement was applied incrementally to the top row of nodes of the finite element mesh, Figure 4. After each increment in applied displacements, the CTOA was calculated using Equation (5) from the Y displacement,  $\delta$ , of the node located at an X distance,  $d = 0.01875$  inches, behind the crack-tip, Figure 7. When the CTOA reached a value of  $\psi_c = 6.1^\circ \pm 0.1^\circ$ , the crack was advanced one element length by releasing the forces of the nodes along the X symmetry line. After each crack extension increment (i.e. node splitting occurrence) the applied displacements were held constant while the CTOA criterion was applied to the new crack-tip node. If the CTOA criterion for the new crack-tip node was not fulfilled, the applied displacements were increased. This procedure was continued until the crack extended to a total length of 0.5 inches. Throughout the entire simulation procedure, the global stress and the corresponding crack extension length and load-line displacements were recorded. In addition, the path integrals, namely, the J-integral and T\*-integral, were calculated as a function of crack extension.

## Results

The applied stress as a function of crack extension predicted using the MADGIC code (solid line) and the ZIP2D code (dashed line) are shown in Figure 8a along with the corresponding experimental data measured in reference 10. As shown in this figure, excellent agreement was obtained between the predictions made using the two finite element codes and the experimental data. Using the CTOA approach in both numerical simulations, the predicted maximum applied stress was within 5 percent of the experimental results.

The applied stress as a function of load-line displacement predicted using the MADGIC code (solid line) and the ZIP2D code (dashed line) are shown in Figure 8b along with the corresponding data measured in reference 21. In general, good agreement was obtained between the predictions made using both finite element codes and the experiments. The predictions essentially traced out the experimental data.

As shown in Figure 8, there was excellent agreement in the predictions made using the MADGIC code and the predictions made using the ZIP2D code despite the differences in the algorithms used to generate crack surfaces in each code and the differences in the element type used. This illustrates the versatility and accuracy of the CTOA approach in numerical methods used to evaluate elastic-plastic fracture. The CTOA is an effective elastic-plastic fracture mechanics parameter.

Figure 9 shows the J-integral evaluated along a far-field path and the  $T^*$ -integral evaluated for several values of  $\zeta$ . The paths of integration were shown earlier in Figures 6b and 6c. As shown in Figure 9, the J-integral continuously increased as the crack extension length increased. Using a value of  $J_{Ic} = 144 \text{ lb/in}$  (J-integral calculated at crack initiation), the value of  $\omega$  is plotted as a function of crack extension in Figure 10. For J-controlled crack growth to be valid for a middle cracked specimen configuration having strain hardening exponent  $n$  approximately 10 as defined in Equation (3), the value of  $\omega$  must be greater than or equal to 100 [13]. The plastic properties of 2024-T3 aluminum alloy are accurately represented using  $n$  approximately 10, but the values of  $\omega$  were all less than 35. The condition for J-controlled crack growth defined in Equation

(2) was not valid, thus, the J-integral approach is apparently not suitable to characterize the elastic-plastic fracture of the problem considered in this study.

In general, the  $T^*$ -integral approaches a constant value as the crack extends for smaller values of  $\zeta$ , Figure 9. As shown in this figure, with increased crack extension, the value and slope of the  $T^*$ -integral decreased as the value of  $\zeta$  decreased. Thus, the  $T^*$ -integral calculated in this study is path-dependent. In a finite element analysis, the value of  $\zeta$  must be small enough to capture the salient features of the crack-tip fields and large enough to insure that numerical errors are not introduced in the calculation of  $T^*$ -integral due to mesh refinement. The smallest value of  $\zeta$  depends on the mesh refinement near the crack-tip. In this investigation, the  $T^*$ -integral evaluated on the smallest value of  $\zeta$  ( $\zeta_1 = 0.015$  inches) approached a constant value of  $T^*$  approximately 315 lb/in, during stable tearing beyond maximum stress, Figure 9. This suggests that the  $T^*$ -integral may be used as an elastic-plastic fracture mechanics parameter for a specific value of  $\zeta$ .

In order to evaluate the  $T^*$ -integral as an elastic-plastic fracture parameter, a comparison is made with the CTOA. This comparison was made by normalizing the  $T^*$ -integral with respect to the CTOA, the radius,  $\zeta$ , and the flow stress  $\sigma_0$ . This normalized  $T^*$ -integral parameter denoted as  $\tau^*$  should be independent of  $\zeta$ . If there is a one to one correspondence between the  $T^*$ -integral and the CTOA, the  $T^*$ -integral curves shown in Figure 9 should overlap using the normalized parameter  $\tau^*$ . However, as shown in Figure 11,  $\tau^*$  is still a function of  $\zeta$ . In this preliminary study, there appears to be a weak relationship between the CTOA and the  $T^*$ -integral evaluated for a specific value of  $\zeta$  during stable tearing beyond maximum stress.

### **Concluding Remarks**

The critical crack-tip opening angle (CTOA) approach was implemented into a damage growth finite element program, MADGIC (Micromechanics Analysis and Damage Growth In Composites), and was used to predict stable tearing in a thin sheet 2024-T3 aluminum alloy specimen containing a middle crack. Predictions of the applied stress as a function of crack extension and applied stress as a function of load-line displacement were compared with

experiments and with similar predictions made using an existing finite element program, ZIP2D. The major differences between the MADGIC code and the ZIP2D code are the algorithms used to generate crack surfaces and the element types employed. Despite these differences, the predictions made using both codes were in good agreement with experimental data demonstrating the versatility and accuracy of the CTOA approach to evaluate elastic-plastic fracture.

The J-integral was evaluated along a far-field path and the  $T^*$ -integral was evaluated for several paths defined by the radius  $\zeta$  from the crack-tip. The value of the  $T^*$ -integral was a function of  $\zeta$ ;  $T^*$ -integral decreased as  $\zeta$  decreased. As the crack length increased, the values of the J-integral continuously increased while the values of the  $T^*$ -integral approached a constant value. There appears to be a weak relationship between the CTOA and the  $T^*$ -integral evaluated along a specific value of  $\zeta$  during stable tearing beyond the maximum stress in the middle cracked 2024-T3 aluminum specimen considered in this preliminary study.

### Acknowledgments

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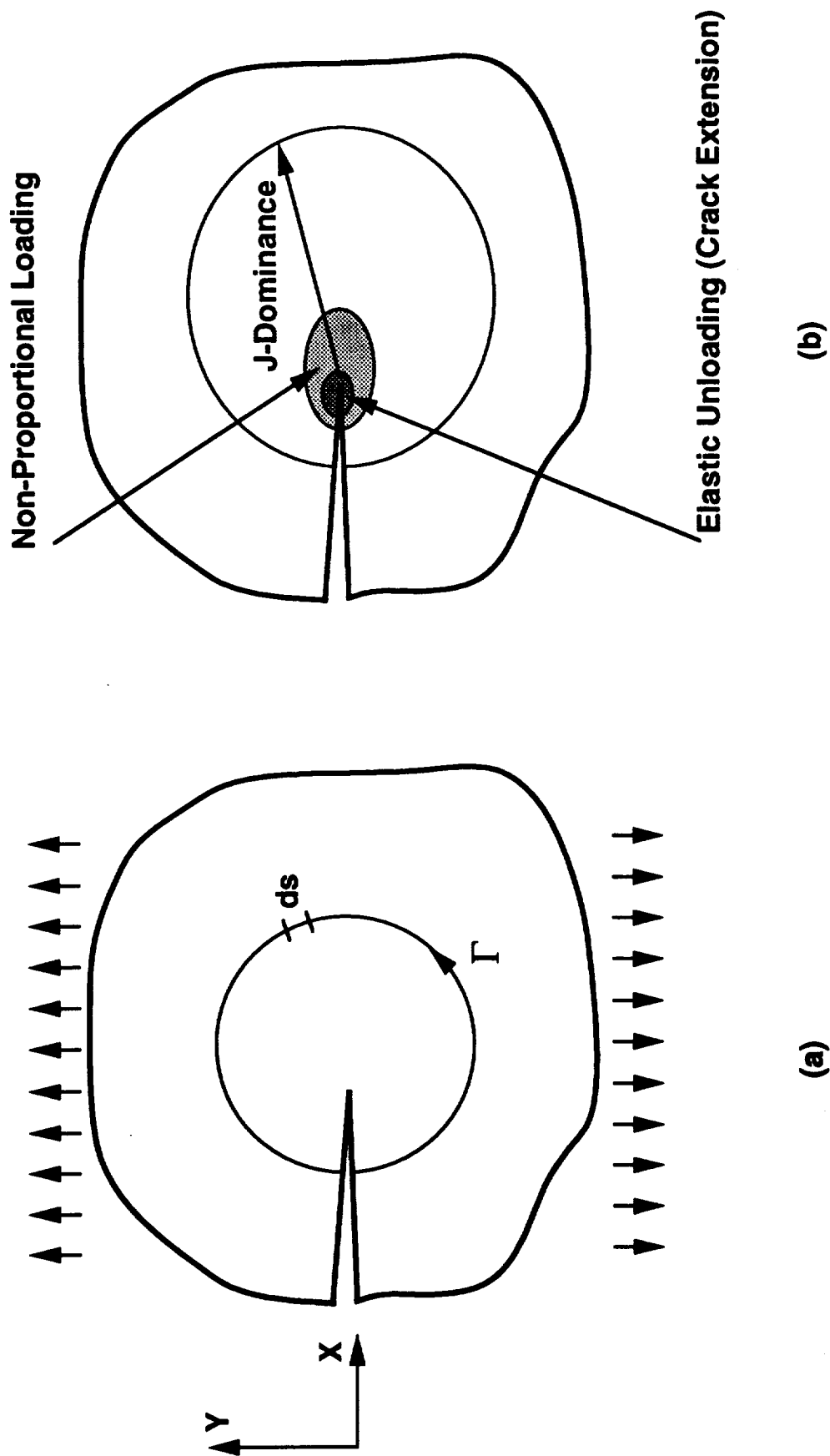


Figure 1. (a) J-integral taken counterclockwise surrounding the crack-tip; (b) Conditions for J-controlled crack growth.

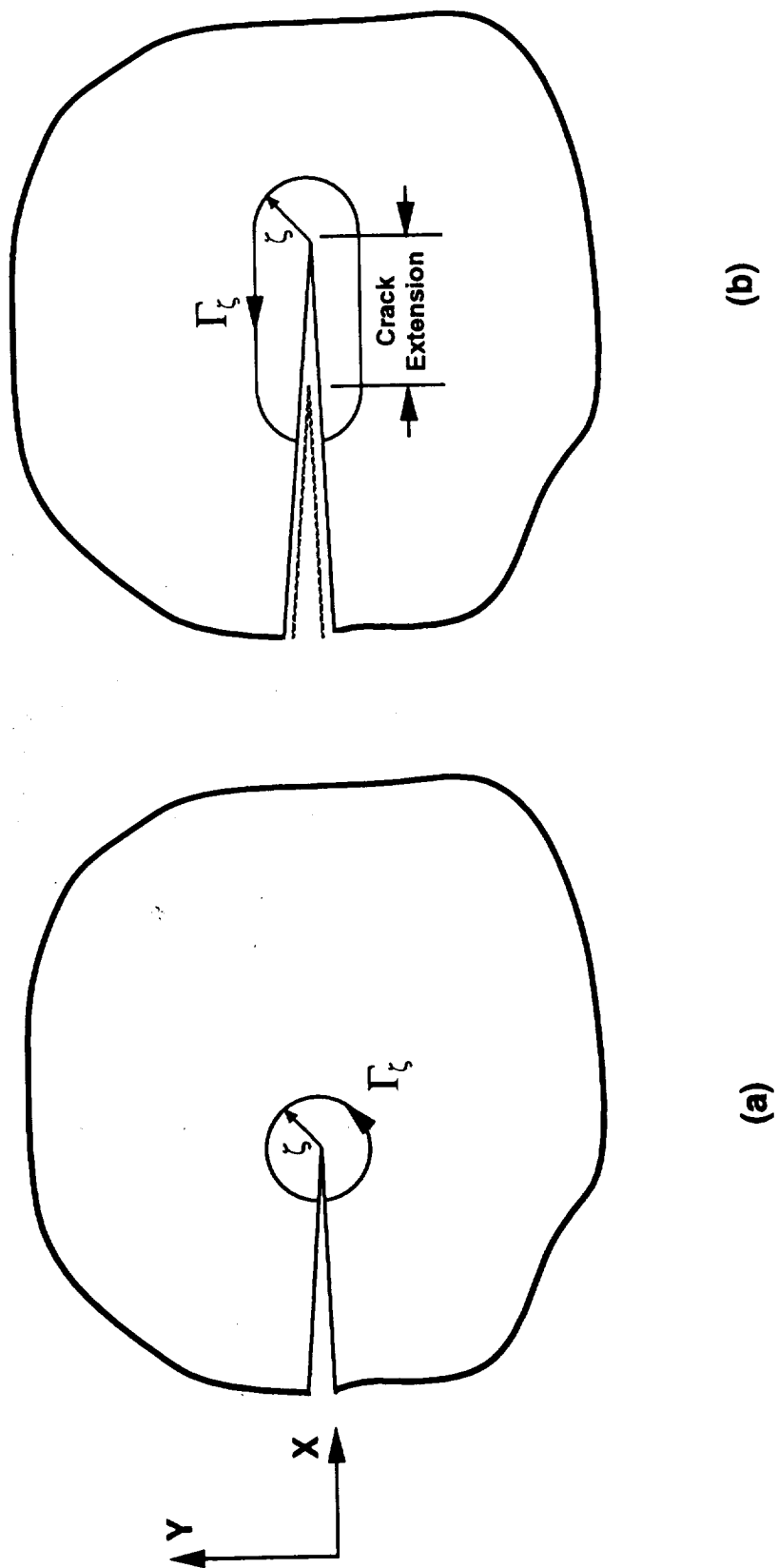


Figure 2. (a) T\*-integral taken on path of radius  $\zeta$  surrounding the crack-tip prior to crack extension; (b) Translation and elongation of integration path as crack grows.

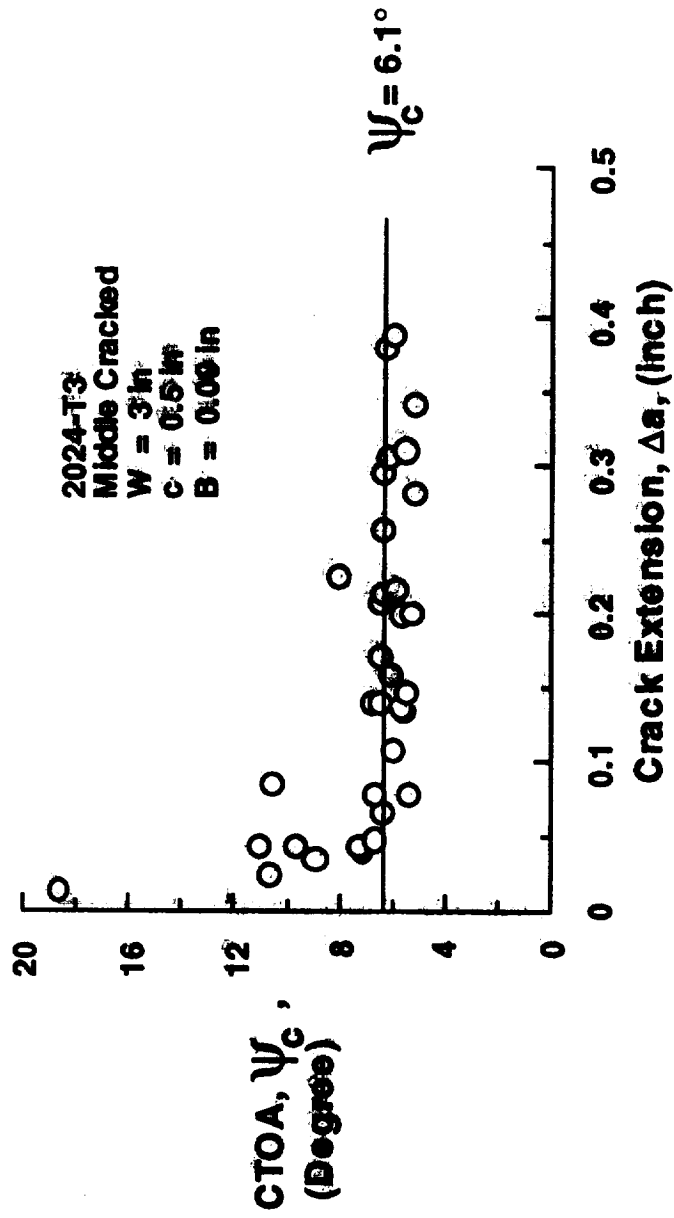
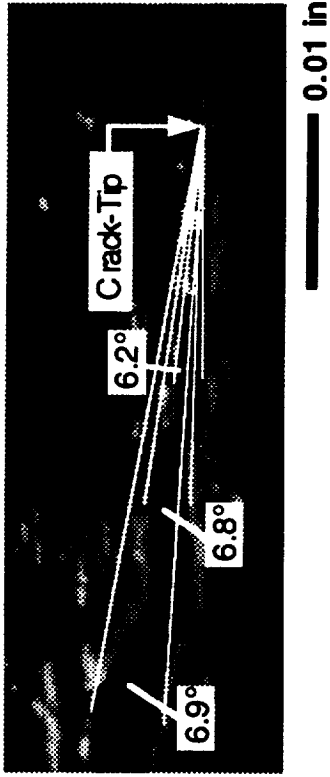


Figure 3. Measured crack-tip opening angles during crack extension in 2024-T3 aluminum [10].



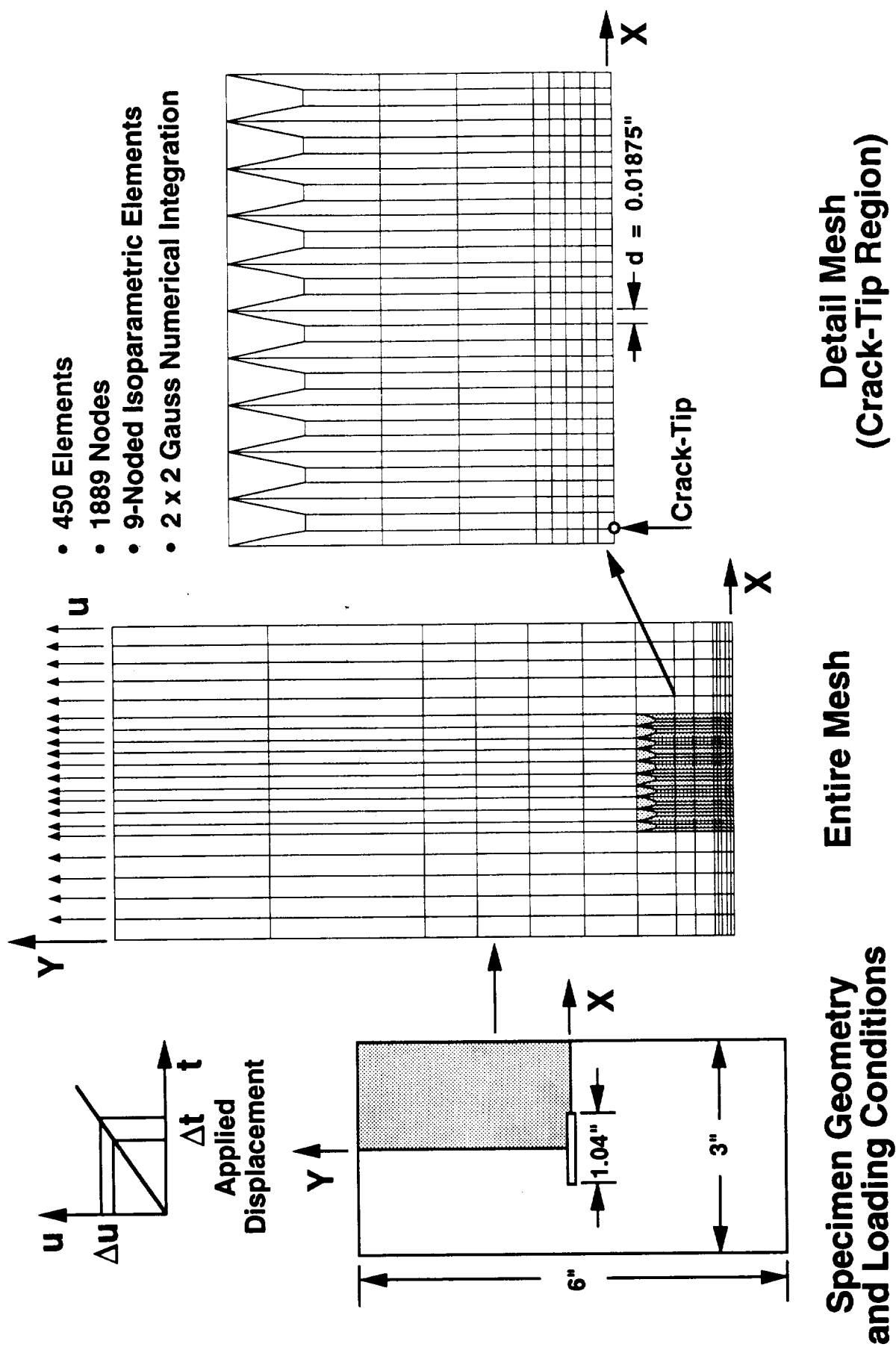
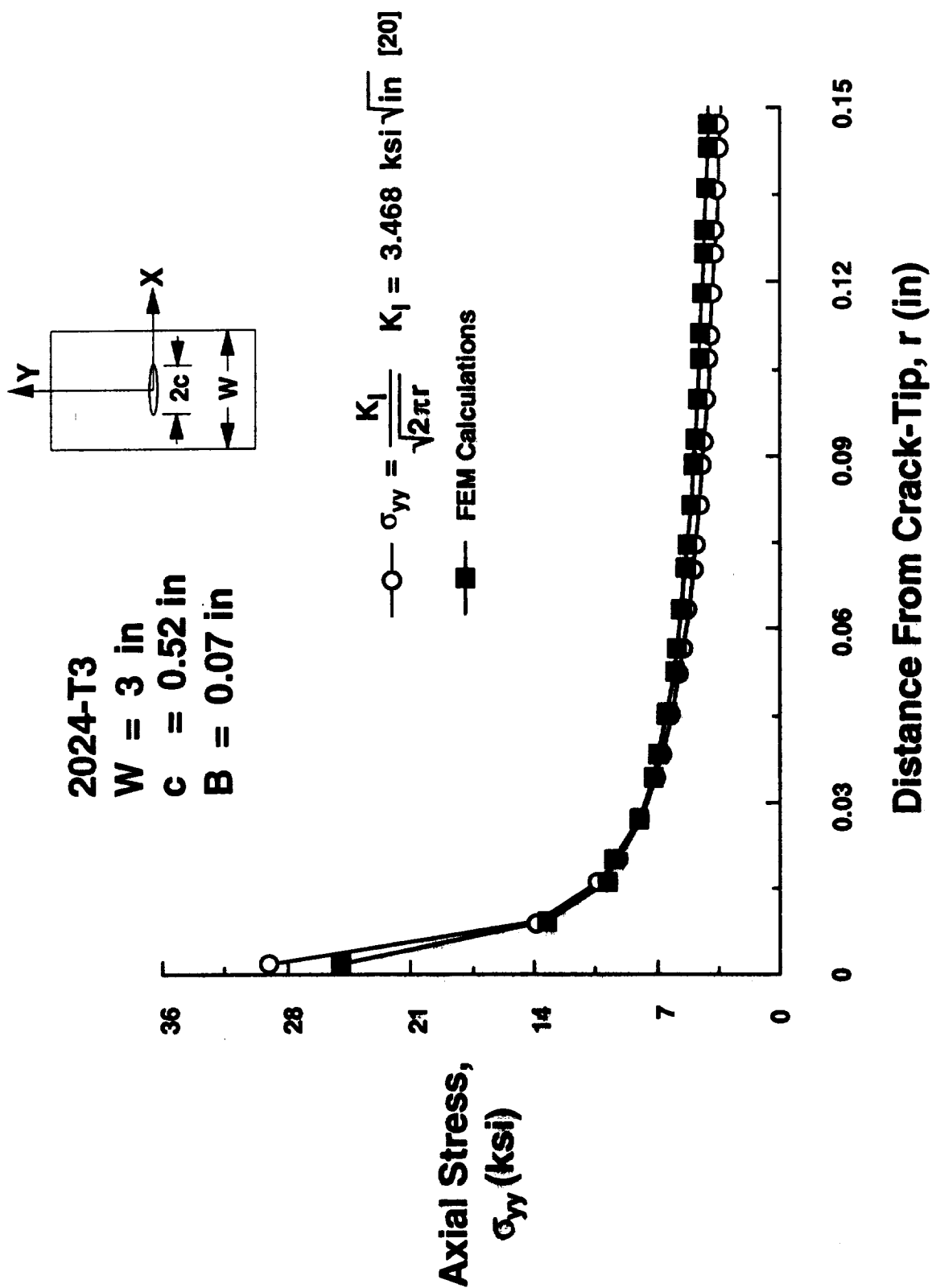


Figure 4. Specimen geometry and finite element model used to simulate stable tearing in 2024-T3 aluminum.



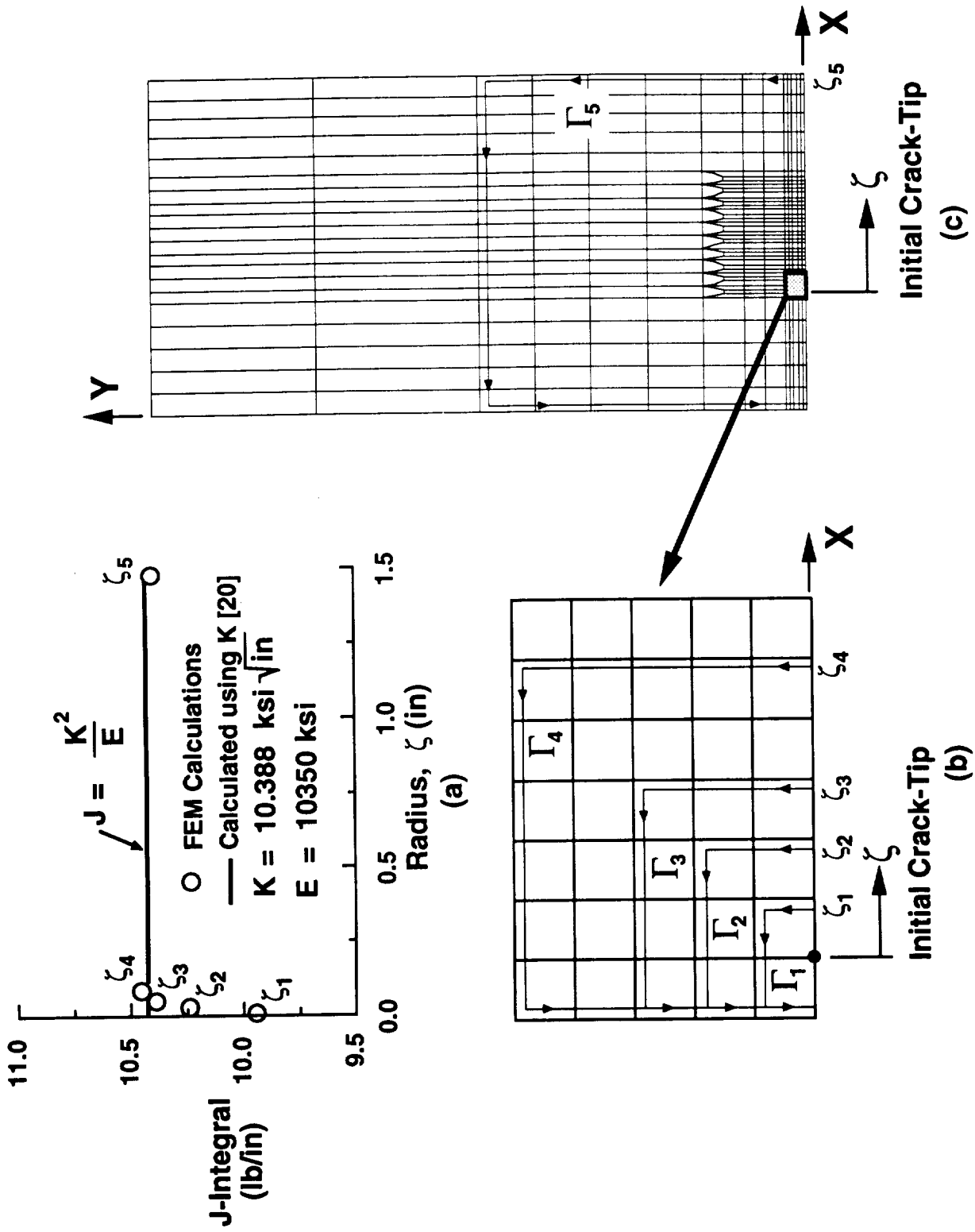


Figure 6. (a) Values of J-Integral taken at several paths,  $\zeta$ . (b) and (c) Location of integration paths.

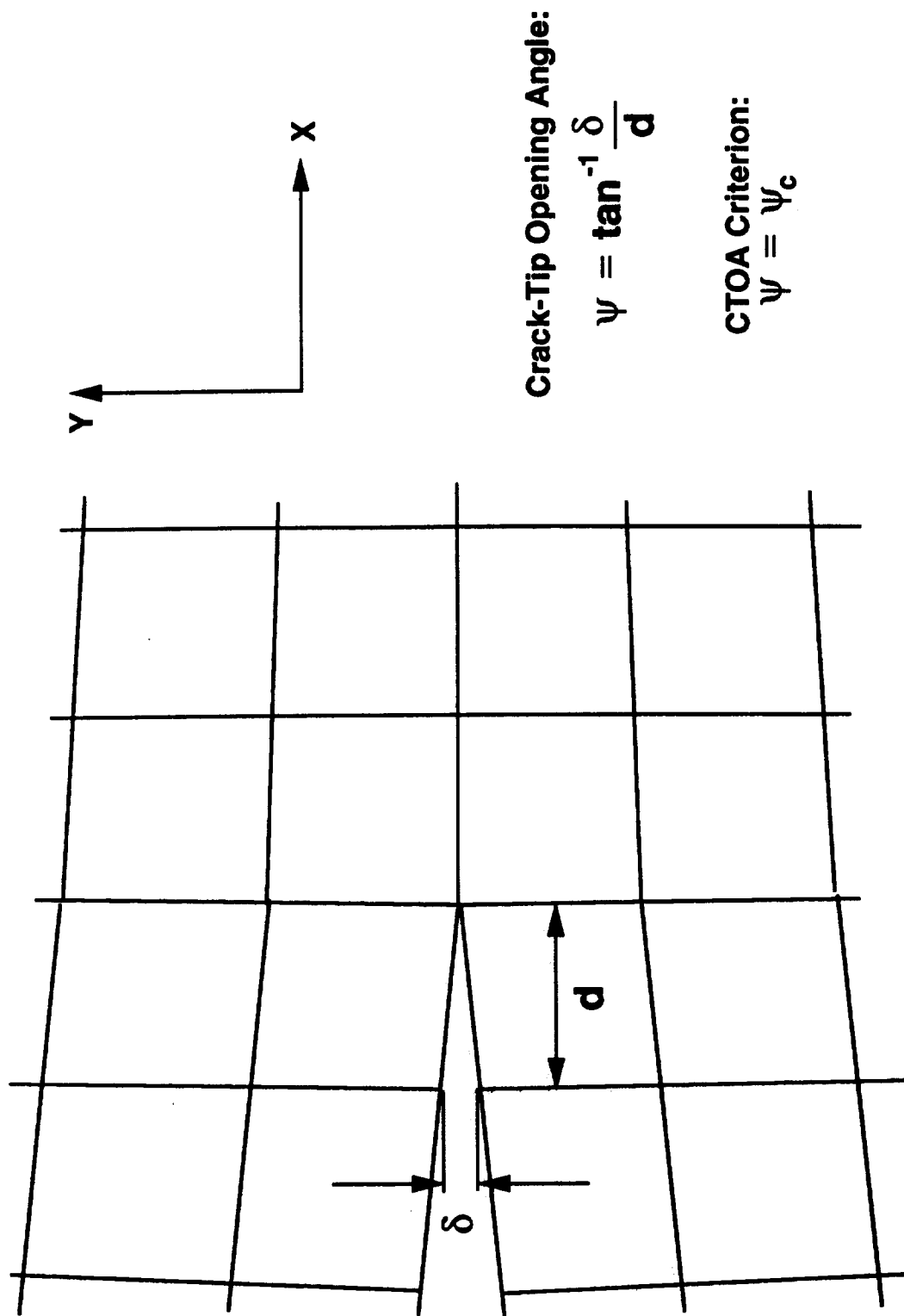


Figure 7. Crack-Tip Opening Angle (CTOA) approach used in MADGIC code.

2024-T3  
 $W = 3 \text{ in}$   
 $c = 0.52 \text{ in}$   
 $B = 0.07 \text{ in}$

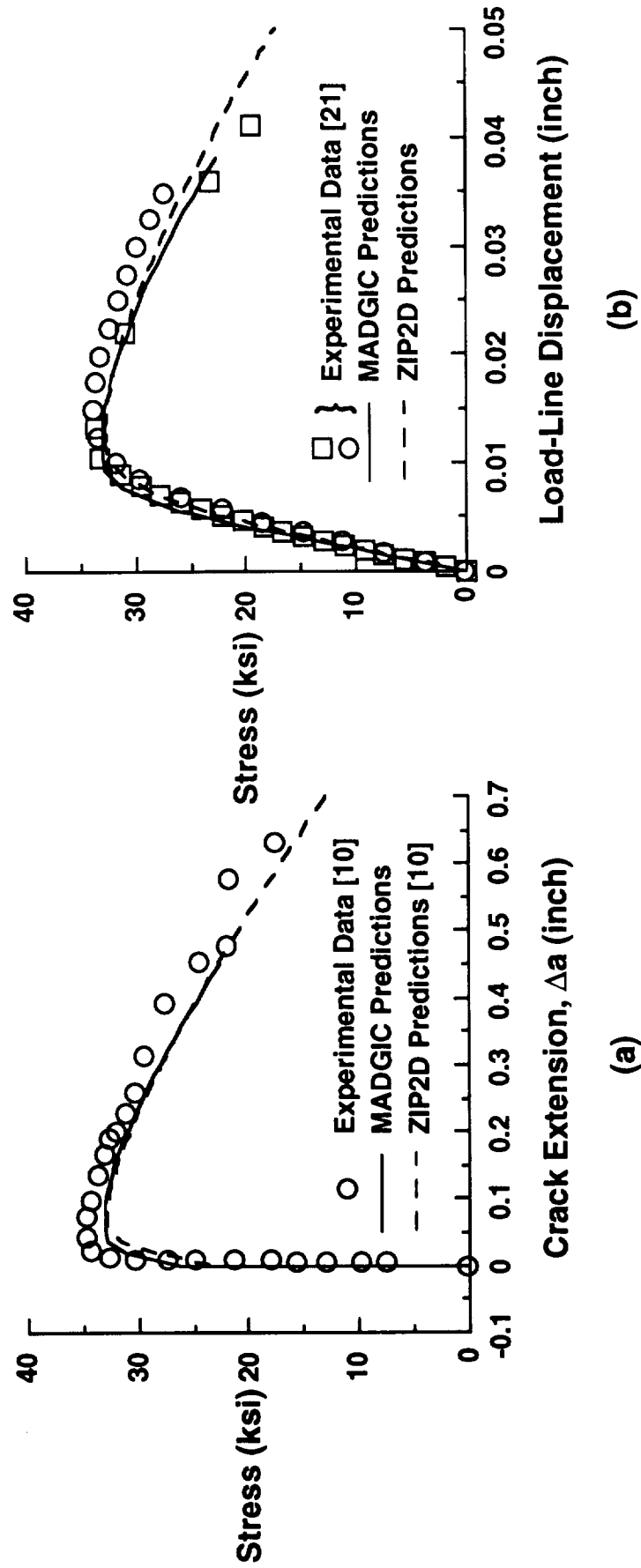


Figure 8. Experiments and predictions made using MADGIC and ZIP2D during stable tearing in 2024-T3 aluminum;  
 (a) Global stress as a function of crack extension length. (b) Global stress as a function of load-line displacement

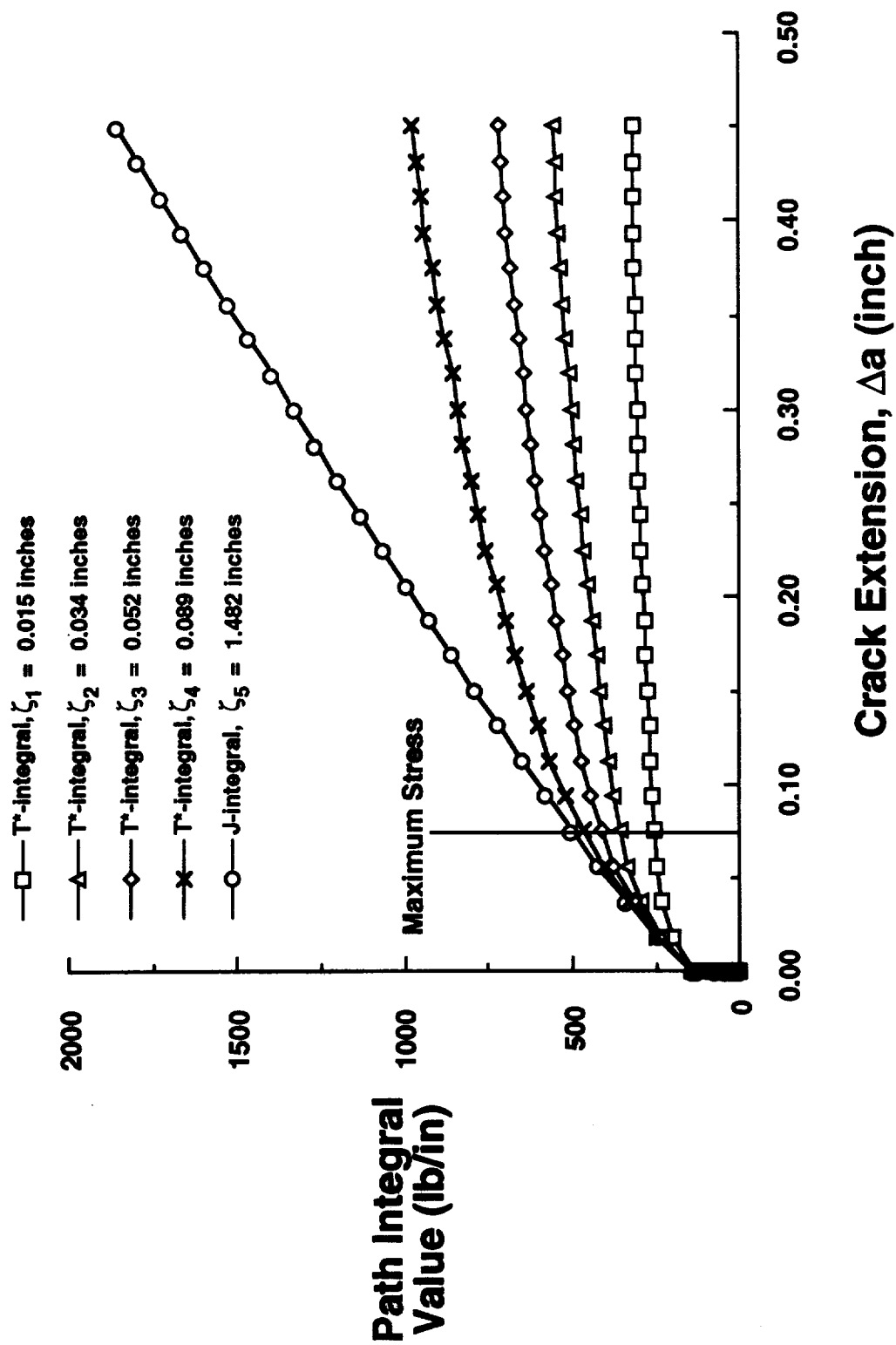


Figure 9. J-integral evaluated in far-field and T\*-integral evaluated at several values of  $\zeta$ .

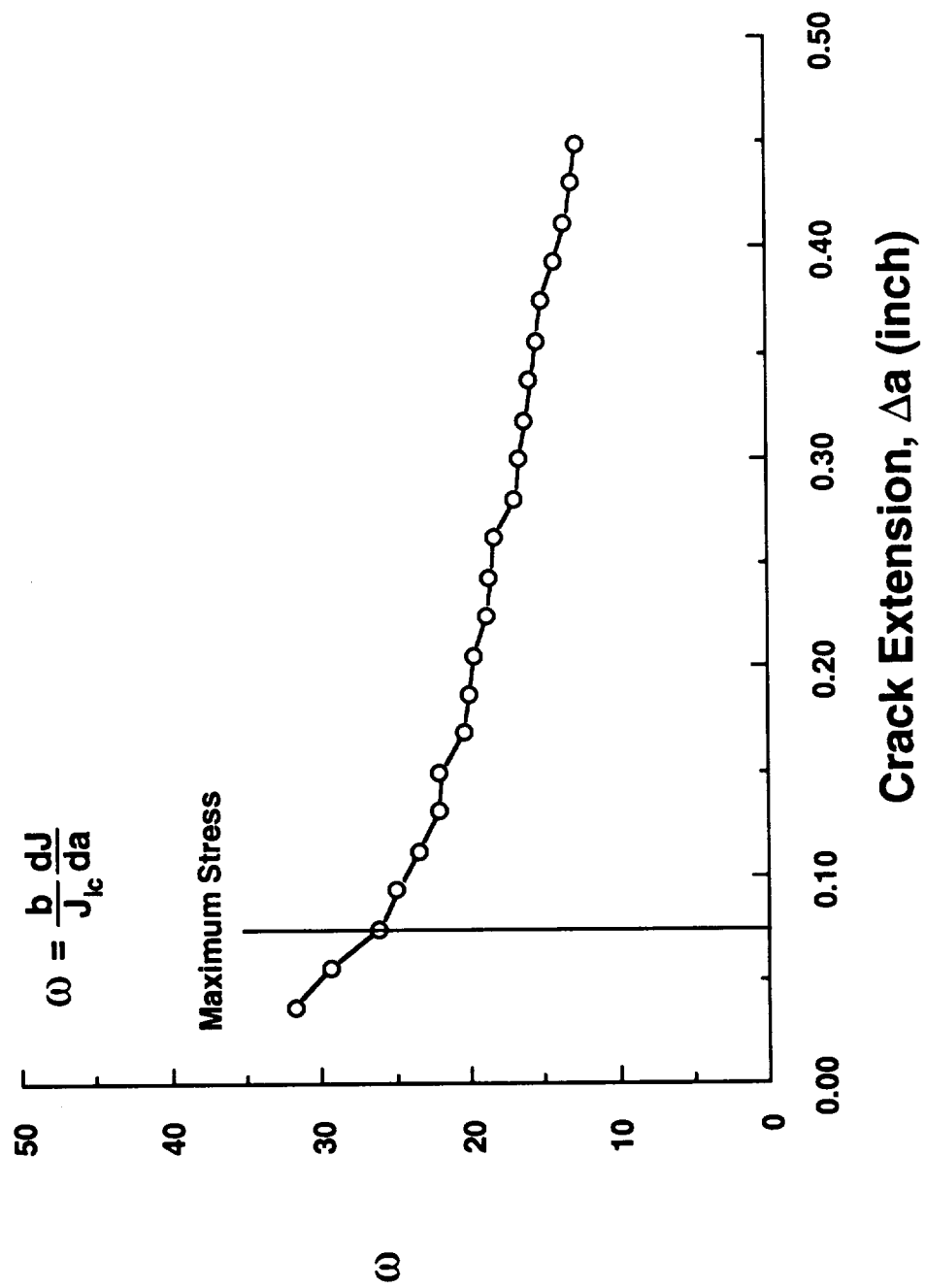


Figure 10. J-controlled crack growth parameter as a function of crack extension.

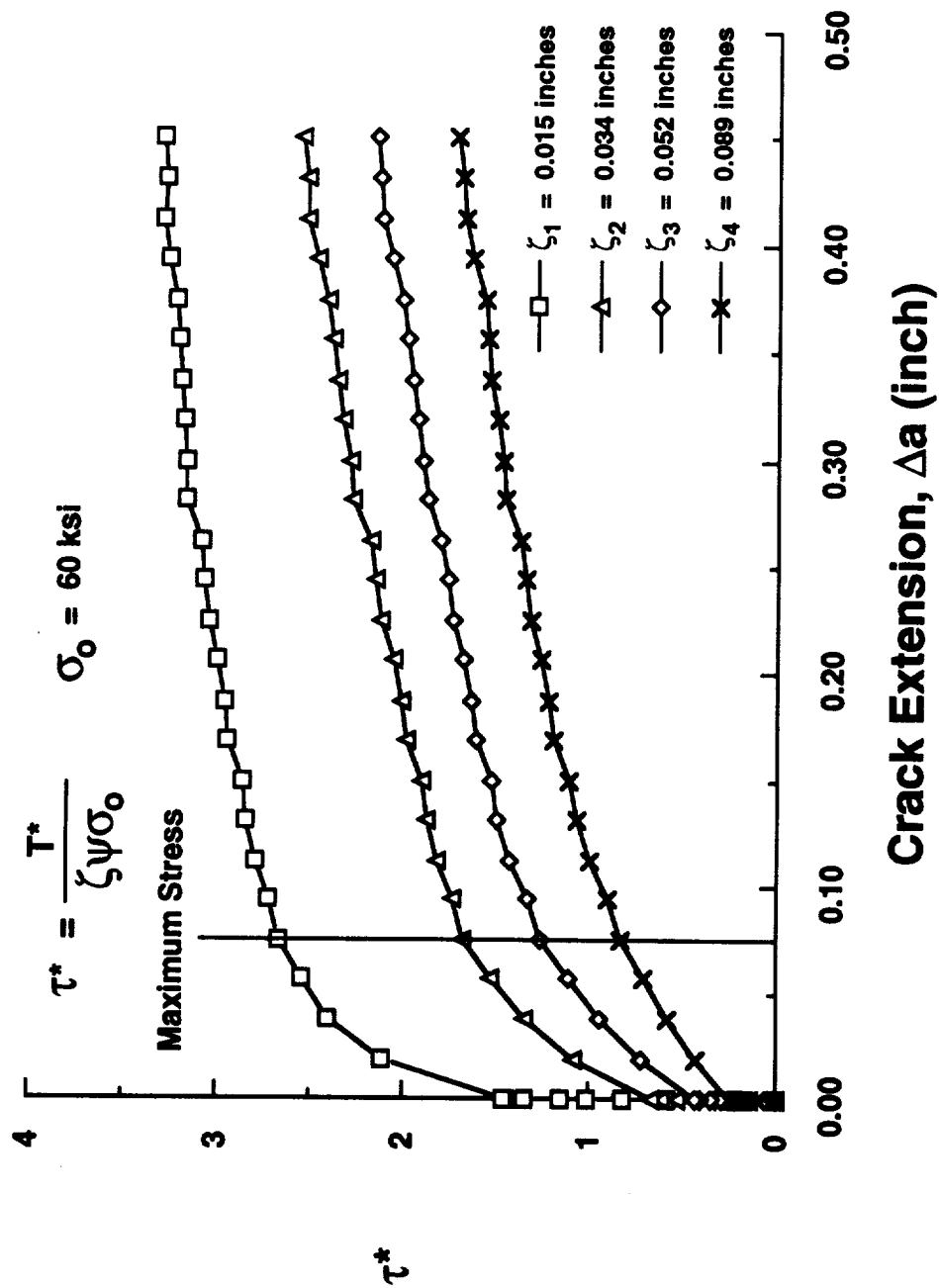


Figure 11. Normalized  $T^*$ -integral as a function of crack extension length.



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13. ABSTRACT (Maximum 200 words)  In this study, the crack-tip opening angle (CTOA) approach was incorporated into a damage growth finite element program, MADGIC (Micromechanics Analysis and Damage Growth in Composites), and was used to predict stable tearing in a middle-crack tension 2024-T3 aluminum alloy specimen. The MADGIC code is a displacement based finite element program implemented with an incremental elastic-plastic algorithm used to model elastic-plastic behavior and a nodal splitting and nodal force relaxation algorithm used to generate crack surfaces. Predictions of the applied stress as a function of crack extension and applied stress as a function of load-line displacement were in good agreement with experiments and with similar predictions made using an existing finite element program, ZIP2D. In addition, path integrals, namely, the J-integral and T*-integral, were also evaluated and compared with the CTOA approach. There appears to be a weak relationship between the CTOA and the T*-integral evaluated on a specific integration path during crack extension beyond maximum applied stress. This study further verifies that the CTOA can be used as an effective elastic-plastic fracture mechanics parameter to predict crack growth.				
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